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1. Find the center and the radius of convergence of:

$$\sum_{n=0}^{\infty} \frac{(4n)!}{2^n (n!)^4} (z + \pi i)^n$$

~~The center of convergence~~

The center of the series is ~~the~~ the series is

$$z + \pi i = 0, \text{ which is:}$$

Center: $-\pi i$

The series could be expressed as:

$$\sum_{n=0}^{\infty} C_n (z + \pi i)^n$$

Daniel Kennedy Villanueva
 Also say to the Cauchy - A is desired ②
 Theorem the radius of convergence is!

$$r = \frac{1}{\limsup_{n \rightarrow \infty} \sqrt[n]{|c_n|}}$$

applying it to our series!

$$r = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{2^n (n!)^4}{(4n)!}}$$

$$r = \lim_{n \rightarrow \infty} \sqrt[n]{2^n} \cdot \sqrt[n]{\frac{(n!)^4}{(4n)!}}$$

$$r = \lim_{n \rightarrow \infty} 2 \cdot \frac{(n!)^{\frac{4}{n}}}{(4n)!^{\frac{1}{n}}}$$

$$r = \lim_{n \rightarrow \infty} \frac{e^{\frac{4}{n} \ln n!}}{e^{\frac{1}{n} \ln (4n)!}} = \lim_{n \rightarrow \infty} \left(e^{\frac{4}{n} \ln n! - \frac{1}{n} \ln (4n)!} \right)$$

using Stirling's approximation

$$(n!) = n \ln n - n + O(\ln n)!$$

Proof of L'Hôpital's rule

$$r = \lim_{n \rightarrow \infty} 2 \left(e^{\frac{4}{n} (n \ln n - n + O(\ln n))} - \frac{1}{n} (4n \ln 4n - 4n) \right)$$

$$r = \lim_{n \rightarrow \infty} 2 \left(e^{4 \ln n - 4 - \frac{O(\ln n)}{n}} - (4 \ln 4n - 4) \right)$$

$$r = \lim_{n \rightarrow \infty} 2 \left(e^{4 \ln n - 4} - 4 \ln 4n - 4 \right)$$

$$r = \lim_{n \rightarrow \infty} 2 \left(e^{-4 \ln 4} \right) = 2 \left(4^{-4} \right) = 2/2^8$$

Therefore!

radius of convergence: 2^{-7}

David Lunnell Williams

2. Find the first four terms of the following Taylor series!

$$e^z (z-2) \text{ centered at } z_0 = 1$$

From the Taylor series of e^x :

$$e^x = \sum_{h=0}^{\infty} \frac{x^h}{h!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots$$

Now substitute the change of ^{center} ~~center~~ and the mapping $z \mapsto z(z-2)$

$$1 + (z(z-2) - 1) + \frac{(z(z-2) - 1)^2}{2} + \frac{(z(z-2) - 1)^3}{6}$$

$$z(z-2) - 1$$

$$z^2 - 2z - 1 = (z-i)^2$$

$$1 + (z-i)^2 + \frac{(z-i)^4}{2} + \frac{(z-i)^6}{6}$$

David Louis Villanueva

3. Find the Power series for terms of the following
rational series!

$$\frac{z+2}{1-z^2}$$

$$\frac{z+2}{1-z^2} = \frac{z+1}{(1-z)(1+z)} + \frac{1}{1-z^2}$$

$$= \frac{1}{1-z} + \frac{1}{1-z^2}$$

from the ^{geometric} series formula!

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$$

$$\frac{z+2}{1-z^2} = \frac{1 + z + z^2 + z^3 + z^4 + z^5 + z^6 + \dots}{1 + z^2 + z^4 + z^6 + \dots}$$

$$\frac{z+2}{1-z^2} = \boxed{2 + z + 2z^2 + z^3 + \dots}$$

Dr. H. K. Villanov

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f. find the Fourier series of the given function, which is assumed to have the period 2π

$$f(x) = \begin{cases} x & \text{if } (-\pi < x < 0) \\ \pi - x & \text{if } (0 < x < \pi) \end{cases}$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx$$

$$\begin{aligned} a_0 &= \frac{1}{2\pi} \left(\int_{-\pi}^0 x dx + \int_0^{\pi} (\pi - x) dx \right) \\ &= \frac{1}{2\pi} \left(\frac{x^2}{2} \Big|_{-\pi}^0 + \pi x - \frac{x^2}{2} \Big|_0^{\pi} \right) = \frac{-\pi^2}{2} + \pi^2 - \frac{\pi^2}{2} \\ &= 0 \end{aligned}$$

$\therefore a_0 = 0$

Daniel Kometz Willmann

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$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} \frac{P(x)}{f(x)} \cos nx \, dx$$

$$T_2 = \frac{1}{\pi} \left(\int_{-\pi}^0 x \cos nx \, dx + \int_0^{\pi} (\pi - x) \cos nx \, dx \right)$$

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$$x \cos nx$$

$$1 \quad \frac{1}{n} \sin nx$$

$$0 - \frac{1}{n^2} \cos nx$$

$$a_2 = \frac{1}{\pi} \left(\frac{1}{n} \sin nx - \frac{1}{n^2} \cos nx \right) \Big|_{-\pi}^0 + \frac{\pi}{n} \sin nx$$

$$a_{2k} = \frac{1}{\pi} \left(\frac{1}{n^2} + \frac{1}{n^2} \right) + \left(\frac{1}{n^2} - \frac{1}{n^2} \right) = 0$$

$$a_{2k-1} = \frac{1}{\pi} \left(\left(-\frac{1}{n^2} - \frac{1}{n^2} \right) + \left(\frac{1}{n^2} - \frac{1}{n^2} \right) \right) = -\frac{4}{n^2 \pi} = \frac{-4}{(2k-1)^2 \pi}$$

$$-\frac{1}{n} \sin nx + \frac{1}{n^2} \cos nx \Big|_{-\pi}^{\pi}$$

Das ist Konstante $f(x) = 1$ hier

⑧

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx$$

$$b_n = \frac{1}{\pi} \left(\int_{-\pi}^0 x \sin nx \, dx + \int_0^{\pi} (\pi - x) \sin nx \, dx \right)$$

0 I

$$\begin{aligned} & \int \sin nx \\ & - \frac{1}{n} \cos nx \end{aligned}$$

$$- \frac{1}{n^2} \sin nx$$

$$b_n = \frac{1}{\pi} \left(-\frac{1}{n} \cos nx - \frac{1}{n^2} \sin nx \Big|_{-\pi}^0 + \left(-\frac{\pi}{n} \cos nx + \frac{1}{n} \cos nx - \frac{1}{n^2} \sin nx \right) \Big|_0^{\pi} \right)$$

$$b_{2k} = \frac{1}{\pi} \left(-\frac{1}{n} + \frac{1}{n} \right) + \left(\frac{1-\pi}{n} - \frac{1-\pi}{n} \right) = 0$$

$$b_{2k+1} = \frac{1}{\pi} \left(-\frac{1}{n} - \frac{1}{n} \right) + \left(-\frac{1-\pi}{n} - \frac{1-\pi}{n} \right) =$$

$$b_{2k+1} = \frac{-4+2\pi}{(2k+1)\pi}$$

$$= \frac{-4+2\pi}{(2k+1)\pi}$$

Q. 10.1. Compute the Fourier series

$$f(x) = \sum_{k=1}^{\infty} \left[\frac{-4 \cos((2k-1)x)}{(2k-1)^2 \pi} + \frac{(2\pi-4) \sin((2k-1)x)}{(2k-1) \pi} \right]$$

Therefore, the Fourier series for the function is:

$$\sum_{k=1}^{\infty} \left[\frac{-4 \cos((2k-1)x)}{(2k-1)^2 \pi} + \frac{(2\pi-4) \sin((2k-1)x)}{(2k-1) \pi} \right]$$